# Performance Evaluation of HF Test Signal Based Sensorless Method for IPMSM in the Presence of Cross-coupling Inductance

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***Abstract*— This paper discusses HF test signal based sensorless methods often used in shaft-sensorless IPMSM drives. Conventional and improved HF test signal injection based sensorless are discussed and some limitation for their practical implementation are exposed. It was found that limitations arise due to the dq axis magnetic saturation and cross-coupling and that are amplified for higher motor loads. Both experiment and simulation results are given.**

***Keywords - IPMSM, HF signal sensorless, cross-coupling***

## NOMENCLATURE

| | |
|---|---|
| $v_d$ ,$v_q$ | – stator *d*- and *q*- axis voltages |
| $i_d$, $i_q$ | – stator *d*- and *q*- axis currents |
| $v_{dh}$ ,$v_{qh}$ | – stator *d*- and *q*- axis high frequency voltage components |
| $i_{dh}$, $i_{qh}$ | – stator *d*- and *q*- axis high frequency current components |
| $\psi_d$, $\psi_q$ | – stator *d*- and *q*- axis fluxes |
| $\psi_f$ | – permanent magnet flux |
| $R_s$ | – stator phase resistance |
| $L_d$, $L_q$ | – stator *d*- and *q*- axis self inductances |
| $L_{dh}$,$L_{qh}$ | – stator *d*- and *q*- axis incremental self inductances |
| $L_{dqh}$,$L_{qdh}$ | – stator *d*- and *q*- axis incremental mutual inductances |
| $\theta_r$, $\theta_r^e$ | – actual and estimated rotor electrical position |
| $\omega_r$ | – actual rotor electrical angular frequency |
| $T_e$ | – electromagnetic torque |
| $P$ | – number of pole pairs |

## I. INTRODUCTION

Shaft-sensorless control of IPMSM motors is attractive for its inherent advantages; such are increased reliability, reduced cost and size of both motor and inverter. But, in order to have stable control of IPMSM the knowledge of actual rotor position is necessary. Most methods for IPMSM rotor position estimation applicable at low speed range are based on high frequency (HF) voltage test signal injection [1]. The HF voltage is injected into the output voltage and machine response is monitored by processing the resulting HF currents. The position estimation is rotor saliency-dependent and in theory does not depend upon rotor speed or back electromotive force (BEMF) magnitude. This technique is therefore effective at zero and low motor speed, where BEMF is null or extremely low. However, it has been shown [2] that HF rotor position estimation error is influenced by *dq*-axis magnetic and cross-coupling saturation. The error increases with the stator current level and in some applications even limits the usage of HF test signal based shaft-sensorless drives. An enhanced HF test signal based method presented in [2] predicts the cross-coupling magnetic saturation and improves the accuracy of the rotor-position estimation.

The results from this paper show that even enhanced HF method does not resolve all the problems associated to magnetic saturation. Section II of this paper presents the mathematical model of IPMSM with saturation and cross-coupling submodels based on measured data [4-8]. Section III presents enhanced HF based sensorless method published in [2]. The enhanced method is closely analyzed in section IV, using both simulation and experimental results which are close match. The main conclusion is that enhanced HF model can cancel the cross-coupling related steady state position error but cannot prevent contamination of position error information. The HF error information used by position estimator remains distorted for all non-zero position errors and can lead to unstable system behavior.

## II. MATHEMATICAL MODEL OF IPMSM WITH SATURATION AND CROSS-COUPLING

Electrical subsystem of IPMSM can be modeled using voltage balance equations (1), flux linkage equations (2) and electromagnetic torque formula (3).

$$\begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_{sd} \\ \psi_{sq} \end{bmatrix} + \omega_r \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_{sd} \\ \psi_{sq} \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} \psi_{sd} \\ \psi_{sq} \end{bmatrix} = \begin{bmatrix} L_d & L_{dq} \\ L_{dq} & L_q \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} \psi_{PM} \\ 0 \end{bmatrix}, \quad (2)$$

$$m_{el} = \frac{3}{2} P\left(\psi_{PM} i_{sq} + \left(L_d - L_q\right) i_{sd} i_{sq}\right), \quad (3)$$

The mechanical subsystem is described by (4):

$$\frac{d\omega_r}{dt} = \frac{1}{J}\left[m_{el} - K_m \omega_r - m_m\right], \quad (4)$$

The *dq* axis magnet saturation can be included in the model by altering $L_d$ and $L_q$ parameters [3] with current level:

$$L_d = L_d\left(i_{sd}, i_{sq}\right),\ L_q = L_q\left(i_{sd}, i_{sq}\right), \quad (5)$$

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The cross coupling magnet saturation is also included in model via flux linkage equations, (2). The cross-coupling inductance $L_{dq}$ also varies with $dq$ axis current levels

$$L_{dq} = L_{dq}(i_{sd}, i_{sq}). \tag{6}$$

Dynamic inductances are neglected in the model.

The state space model of IPMSM is given in (7):

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \frac{1}{p}\left(-\omega_r[A_1]\cdot\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + [A_2]\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + [B]\begin{bmatrix} u_{sd} \\ u_{sq} \\ \omega_r\psi_{PM} \end{bmatrix}\right) + [C]\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}, \tag{7}$$

where: $[A_1] = \begin{bmatrix} \frac{L_{dq}}{L_d} & \frac{L_q}{L_d} \\ \frac{L_d}{L_q} & \frac{L_{dq}}{L_q} \end{bmatrix}$, $[A_2] = \begin{bmatrix} \frac{R_s}{L_d} & 0 \\ 0 & \frac{R_s}{L_q} \end{bmatrix}$,

$[B] = \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & -\frac{1}{L_q} \end{bmatrix}$, $[C] = \begin{bmatrix} 0 & -\frac{L_{dq}}{L_d} \\ -\frac{L_{dq}}{L_q} & 0 \end{bmatrix}$

For tested motor, parameters $L_d$, $L_q$ and $L_{dq}$ are obtained using the procedure given in [3]. The different current levels are used, and overall results are presented in the figures 1-3. One can find that all the inductances are influenced by magnetic saturation, but $q$ axis inductance descend is the most noticeable.

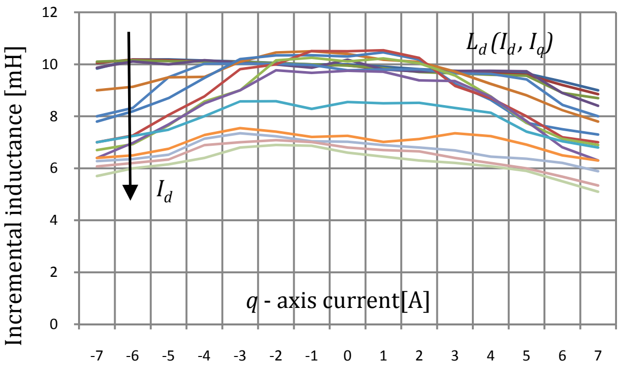


Figure 1. Measured incremental $d$-axis self-inductance

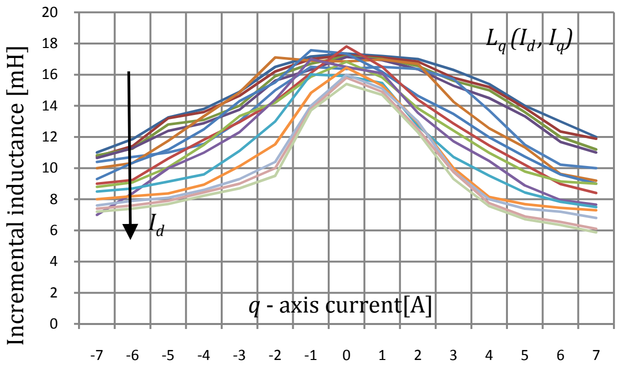


Figure 2. Measured incremental $q$-axis self-inductance

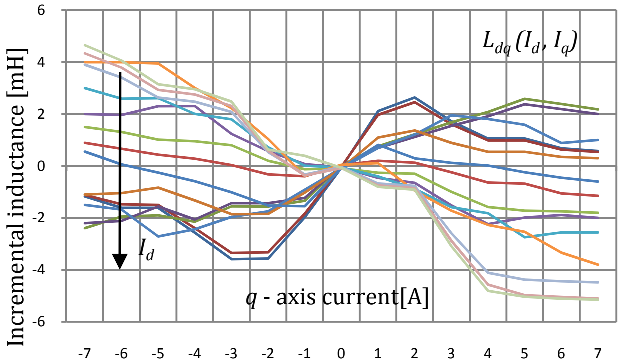


Figure 3. Measured incremental $dq$ cross-coupling inductance

## III. HF TEST SIGNAL BASED ROTOR POSITION ESTIMATION METHOD WITH ELIMINATION OF CROSS-COUPLING INFLUENCE

HF test signal based technique can be used for initial rotor position detection of any IPMSM, or for parallel speed and position estimation during the normal operation of shaft-sensorless IPMSM. The estimation is based on the injected HF test signal. The HF signal can be injected as pulsation in the estimated $dq$ reference frame or as revolving carrier in $\alpha\beta$ frame. In both cases the rotor position error is measured by monitoring the machine HF current response. If the $d$ axis used in controller is not aligned with actual rotor magnet axis the injection of HF voltage test signal into that axis will result in appearance of HF current signal in orthogonal, controller's $q$ axis [9]. The HF current signal in $q$ axis will disappear if controller's $d$ axis matches rotor magnet axis. Therefore, that HF current signal can be used as position error information. The HF voltage amplitude is usually chosen as constant and needs to be large enough to give measurable HF current signal for given machine inductance. The HF signal frequency ($\omega_{HF}$) selection is tradeoff between the needed HF current signal amplitude, allowed acoustic noise level and processor power needed for digital signal processing. HF current signal must be separated from the fundamental frequency current using band-pass filters. In that way, influence of current controller dynamics can be avoided.

The HF test signal rotor magnet position estimation method is closely explained in the figure 4. Due to the position error, injected HF voltage vector in controller's $d$ axis will have components in both $d$ and $q$ axis of real rotor magnet. Those components create corresponding HF current signals. If the $d$ and $q$ rotor axis inductances, $L_d$ and $L_q$, are different, as it is the case in IPMSM, the resulting $d$ and $q$ HF currents will be with different magnitude. That will create the shift of resulting HF current vector which now also has the non-zero component in controller's $q$ axis. That current vector component can be measured and used as signal of position error. As it is shown on figure 4, the positive position error results in positive $q$ current component. On the other hand, negative position error results in negative $q$ current component [9].

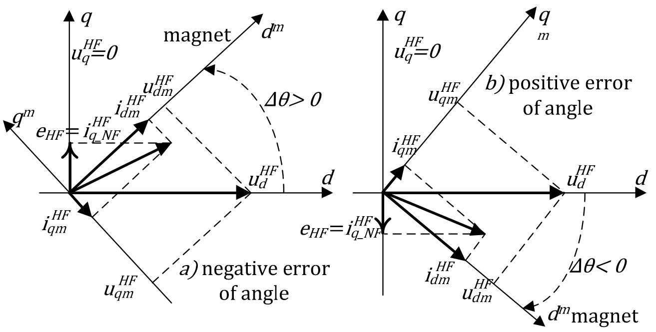


Figure 4. HF voltage impressed in d axis, and an explanation of the appearance of HF current in *q*-axis of the controller.

This information in $q$ axis of HF current signal is used by an algorithm for the rotor position estimation as shown on Fig. 5 and Fig. 6. In the $d$ axis is injected HF test signal with the $\omega_{HF}$ frequency (Fig. 5), while band-pass filter (BPF) in $q$ axis separate HF current signal from the fundamental frequency (Fig. 6). Demodulated HF error signal $e_{HF}$, drives the linear proportional-integral controller of $dq$ angular frequency $\omega_{dq}$. The estimated angle of $dq$ system $\theta_{dq}$ is then obtained by integrating the estimated $\omega_{dq}$ signal. If the error of angle $\Delta\theta_{dq}$ is negative, the PI regulator increases the $\omega_{dq}$ frequency which moves $\theta_{dq}$ towards actual magnet position $\theta_{dqm}$. And vice versa. The angle $\theta_{dq}$ is used in Park transformations and thus moves $dq$ DSP system. If estimated and magnet position are match, HF current disappear from $q$ axis, error signal is zero and proper $\omega_{dq}$ is maintained with integral action of $\omega_{dq}$ regulator.

Due to the load dependent current level, only second mode can be influenced by magnetic saturation. In the presence of load the cross-coupling inductance appears and must be included in the model. When only the high-frequency signal-injection components are considered, equation (1) can be showed as:

$$\begin{bmatrix} v_{dh} \\ v_{qh} \end{bmatrix} = \begin{bmatrix} L_{dh} & L_{dqh} \\ L_{qdh} & L_{qh} \end{bmatrix} p \begin{bmatrix} i_{dh} \\ i_{qh} \end{bmatrix} \tag{8}$$

Equation (8) can be transformed from the rotor-position reference frame ($\theta r$) to the estimated rotor-position reference frame ($\theta_r^e$) by using the following transformation matrix $T(\Delta\theta)$,

$$T(\Delta\theta) = \begin{bmatrix} cos(\Delta\theta) & sin(\Delta\theta) \\ -sin(\Delta\theta) & cos(\Delta\theta) \end{bmatrix} \tag{9}$$

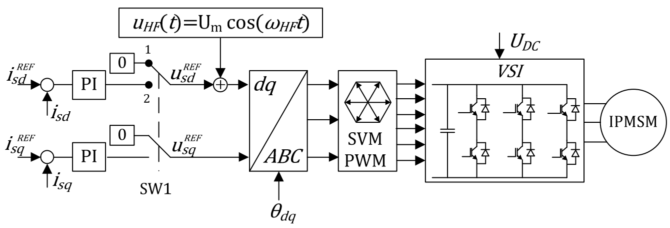


Figure 5. HF voltage injection test signals in *d* axis.

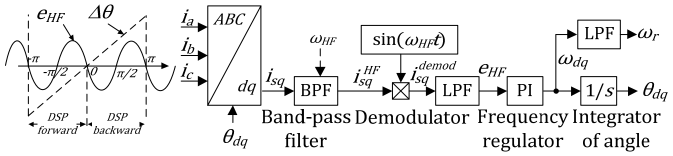


Figure 6. Algorithm to estimate the rotor position of IPMSM based on HF test signal.

where $\Delta\theta$ is the error in the estimated rotor position, i.e., $\Delta\theta = \theta_r^e - \theta_r$ is the difference between the estimated rotor position and the actual rotor position.

$$\begin{aligned} \begin{bmatrix} u_{dh}^e \\ u_{qh}^e \end{bmatrix} &= T(\Delta\theta) \begin{bmatrix} u_{dh} \\ u_{qh} \end{bmatrix} \\ &= T(\Delta\theta) \begin{bmatrix} L_{dh} & L_{dqh} \\ L_{dqh} & L_{qh} \end{bmatrix} T^{-1}(\Delta\theta) p \begin{bmatrix} i_{dh} \\ i_{qh} \end{bmatrix} \end{aligned} \tag{10}$$

$$\begin{bmatrix} u_{dh}^e \\ u_{qh}^e \end{bmatrix} = \begin{bmatrix} L_{avg} - x & y \\ y & L_{avg} + x \end{bmatrix} p \begin{bmatrix} i_{dh} \\ i_{qh} \end{bmatrix} = \begin{bmatrix} u_{sig} \\ 0 \end{bmatrix} \tag{11}$$

where:

$$x = \hat{L}_{diff} cos(2\Delta\theta + \theta_m), \qquad y = \hat{L}_{diff} sin(2\Delta\theta + \theta_m)$$

$$L_{avg} = \left(L_{dh} + L_{qh}\right)/2 \;\; L_{diff} = \left(L_{qh} - L_{dh}\right)/2 \tag{12}$$

$$\hat{L}_{diff} = \sqrt{{L_{diff}}^2 + {L_{dqh}}^2}\; \theta_m = arctan\left(\frac{L_{dqh}}{L_{diff}}\right) \tag{13}$$

The high-frequency voltage signal, $u_{sig}$=V$_{inj}$ $sin(2\pi f_{HF}t)$, is applied to the $d$-axis. Due to the position error, the high-frequency $q$-axis currents appears in the estimated rotor-position reference frame,

$$i_{qh}^e = -\frac{u_{sig}}{p\left({L_{avg}}^2 - {\hat{L}_{diff}}^2\right)} \hat{L}_{diff} sin(2\Delta\theta + \theta_m) \tag{14}$$

The essence of the HF test signal position estimation method is to force this $i_{qh}^e$ current to zero and therefore to cancel the rotor position error. However, as (14) shows, due to the cross-coupling ( non zero $\theta_m$ term) the $i_{qh}^e$ current will appear even if there is no position error, with $dq$ frame exactly in parallel with the magnet. In this case, false error signal exists and after the demodulation further drives the $dq$ system to erroneous position. This error can be predicted,

$$\Delta\theta = \theta_r^e - \theta_r = -\theta_m/2 \approx \frac{1}{2} arctan\left(\frac{2L_{dqh}}{L_{dh} - L_{qh}}\right) \tag{15}$$

where $\theta_r^e$ and $\theta_r$ are the estimated and actual rotor positions, respectively. Clearly, the rotor-position error will be zero only when $L_{dqh}$ = 0, while the stronger the cross coupling between the $d$ and $q$-axes, the larger will be the error.

Method that cancel above-mentioned position error is proposed in [2]. The method is based on Maclaurin series approximation of sine and cosine functions that is valid for small position errors. For small position errors only, both $d$ and $q$ HF current can be approximated as

$$i_{dh}^e = \frac{u_{sig}}{p\left({L_{avg}}^2 - {\hat{L}_{diff}}^2\right)} \left(L_{qh} + 2L_{dqh}\Delta\theta\right) \tag{16}$$

$$i_{qh}^{e} = \frac{u_{sig}}{p\left(L_{avg}{}^{2} - \hat{L}_{diff}{}^{2}\right)}\left(-L_{dqh} - 2L_{diff}\Delta\theta\right) \quad (17)$$

The method suggest the position error cancelation by making new current error signal that is linear combination of $i_{dh}^{e}$ and $i_{qh}^{e}$. If one multiplies (16) by $L_{dqh}/L_{qh}$ and adds is to (17) resulting HF current signal is

$$i_{qh}^{e} + \lambda i_{dh}^{e} = \frac{u_{sig}}{p\left(L_{avg}{}^{2} - \hat{L}_{diff}{}^{2}\right)}\left(-2\lambda L_{dqh} - 2L_{diff}\right)\Delta\theta \quad (18)$$

New HF current made as combination of $d$ and $q$ currents (18) does not have $\theta_m$ term and disappears when angle error is zero ($\Delta\theta = 0$). As result, the $\omega_{dq}$ regulated is fed by zero error when $dq$ and magnet positions are match and there is no steady state position error. Parameter $\lambda = L_{dqh}/L_{qh}$ is defined as coupling factor [2], which can either be calculated from the machine parameters or deduced experimentally. In practical implementation the signal $i_{qh}^{e} + \lambda i_{dh}^{e}$ is demodulated with $i_{dh}^{e}$ and forced to zero. When the cross coupling is sufficiently small compared with $L_{qh}$, i.e., $L_{dqh} \approx 0$ and, hence, $\lambda = 0$, the proposed sensorless is identical to the conventional signal injection based sensorless method.

## IV. COMPARASION OF IPMSM POSITION ESTIMATION RESULTS USING ORIGINAL AND IMPROVED HF SENSORLESS METHOD

In this chapter performance of both HF test signal based shaft-sensorless methods are closely analyzed. The 8-pole IPMSM with significant cross-coupling inductance is used. In order to effectively demonstrate the impact of cross-coupling and some limitation of both methods, the simulation and experimental results have been obtained. During the IPMSM model simulations the look-up tables based on parameters given in figures 1-3 are used.

On figures 7 – 9 simulation results are given. Figure 7 and 8 shows conventional and improved HF injection based sensorless method results for locked rotor and with $dq$ frame position artificially moved from $-\pi$ to $+\pi$. The locked rotor stator current is regulated at three levels, 0A, 3A and 6A. Both figures show the demodulated HF error signal as function of rotor position error when modeled mutual inductance is different from zero, $L_{dq} \neq 0$, figure 3. Figures 7 and 8 demonstrate the main problem during the practical implementation of HF test signal based sensorless, the HF error signal that drives regulator gets heavily distorted, especially at high loads. The enhanced HF test signal based sensorless cannot help, it results in similar HF error signal distortions for all non zero position errors. The distortion is high, figure 7-8, and HF test signal based estimator can even go to positive feedback for position errors larger than $\pi/4$.

But, as suggested in the paper [2], the improved method does eliminate the steady state position errors. One can find, figure 9, zoomed HF error signal and rotor position error around the zero. Figure shows that improved HF error signal has the same zero crossing with position error signal, which implies that there will be no steady state error in the estimated rotor position.

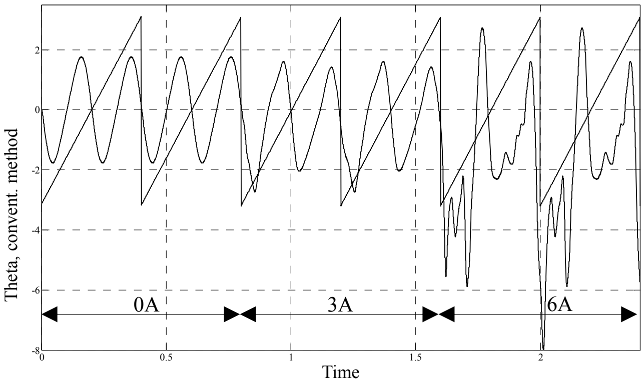


Figure 7. Demodulated HF error signal as function of rotorposition error, conventional method, $L_{dq} \neq 0$

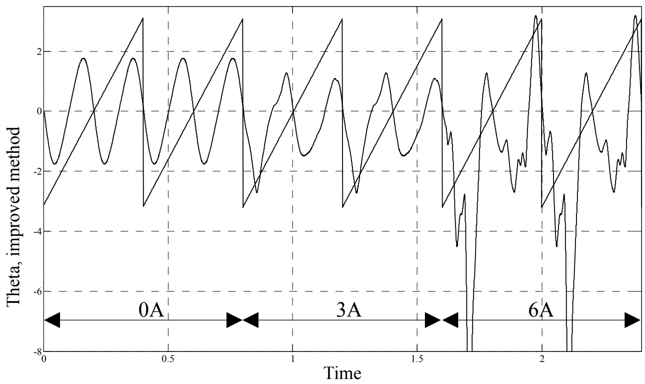


Figure 8. Demodulated HF error signal as function of rotorposition error, improved method, $L_{dq} \neq 0$

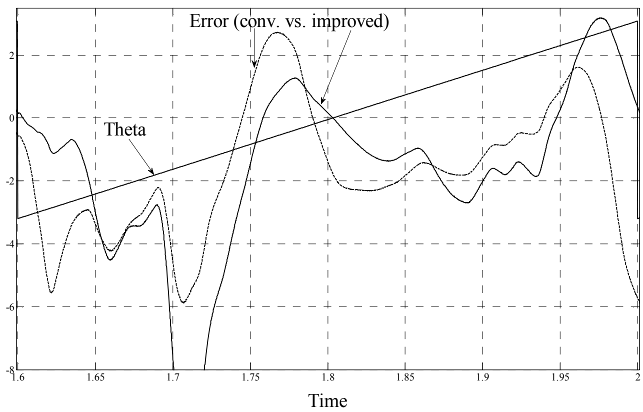


Figure 9. Demodulated HF error signal as function of rotorposition error, conventional (dashed) and improved method, $L_{dq} \neq 0$

The proposed HF signal injected based method and his limitation was verified by using an experimental setup consisting vector controlled 1kW IPMSM machine. Motor was kept at stand still with lock rotor bracket. The magnet and encoder position was preset to zero. HF test signal, amplitude 0.2A and frequency 250Hz, was injected in slowly rotating estimated rotor position, DSP $d$-axis. Shown demodulated rotor position error signal is calculated using conventional and improved method. The data from DSP was transferred into PC via fast GUI interface.

Fig. 10 and 11. show the demodulated HF error signal as function of rotor position error, obtained experimentally by conventional and improved method. Fig 12. shows the same data but zoom, the same as Fig 9, and proves that improved method eliminate the steady state position error.

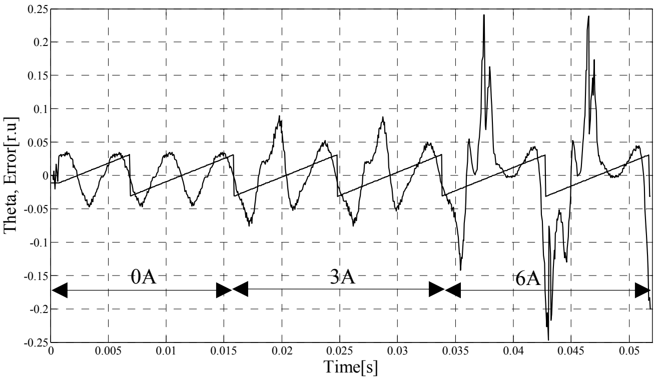


Figure 10. Demodulated HF error signal as function of rotor position error, experimental results, conventional

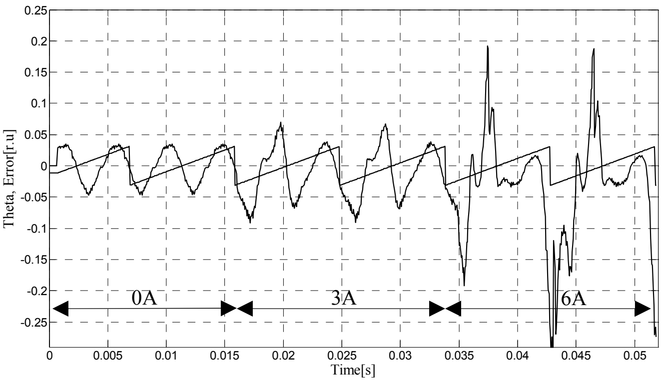


Figure 11. Demodulated HF error signal as function of rotor position error, experimental results, improved

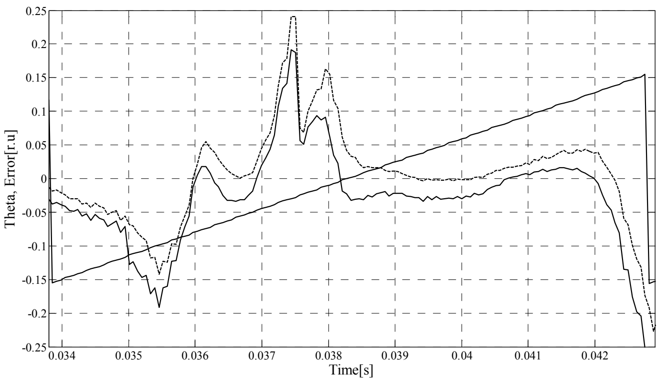


Figure 12. Demodulated HF error signal as function of rotor position error, experimental results, improved and conventional (dashed)

One can find significant similarity between simulation and experimental results. Both type of results show that in [2] proposed algorithm correction does not fully cancel the cross-coupling influence. While steady state error is canceled the HF error curve shape stays distorted. That distortion can lead to HF position regulator positive feedback with unpredictable results, especially for high position error.

## V. CONCLUSION

There are two conclusions to be made. First, the steady state rotor position error due to axis magnetic cross–coupling can be canceled using the enhanced HF test signal method proposed in [2]. One can find that HF position error signal goes to zero when estimated *dq* frame position matches the rotor magnet position. Second, the cross-coupling related distortion of HF error signal for position errors different than zero, especially valid for high loads, cannot be canceled. The improved HF shaft-sensorless method suggested in [2] cannot help and results in similarly distorted position error signal. The reason for that distortion is the approximation used in (16) and (17), valid for position errors close to zero only. The distortion of HF error signal can force the estimated position regulator into positive feedback, especially for high loads and position errors higher than 45 degs. This cannot be ignored in practical implementation of HF method, for example during high accelerations/decelerations, and some new method of cross coupling cancelation should be found. It would be practical to find an error cancelation method that cancels the axis magnetic cross-coupling influence on to position error signal at least in the range of position error $\pm\pi/4$.

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