Impedance Determination in Frequency Domain for Energy Cables by FEM and TLM

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Abstract—**This article deals with the study of the description of phenomena occurring in high frequency domain in two-wires Unshielded and Shielded Energy Cables. First, the finite element model of the cable is presented for calculating the lumped-parameters which depend on the frequency range, especially for the resistance with the phenomenon of skin effect and proximity effect. Then the Transmission Line Method (TLM) is used to determine the impedance of the cable according to the frequency. The numerical results are compared with the experimental results.**

Keywords- Finite element method; frequency-domain analysis; power cable; transmission line models; skin and proximity effect.

I. INTRODUCTION

Modeling cables and power lines are studied over several decades [1]. Currently, more isolated generators or generating farms are connected to power networks. These new sources of electric energy are connected to the network by means of cables and power electronic converters [2], [3]. To avoid disruptions cause by negative resonance frequencies it is necessary to precisely model the behavior of the cable. However, the modeling of energy cables presents some difficulties. This is due to several factors. The properties of materials, thicknesses of insulation and shielding are not fully known. In addition, electrical wires and frame are twisted (sometimes in opposite sense). These physical parameters are insufficient to model a cable in the frequency domain, it is necessary to take into account the electromagnetic phenomena such that the skin effect and the proximity effects. Software, such as EMTP, offers models of cables based on an analytical approach to determine the parameters (lumped or distributed) of cable [4]. Usually with this approach, the skin and the proximity effects are often idealized.

To correctly model these both effects depending highly on the characteristics of the materials and also on the geometry, we propose to use the Finite Element Method (FEM) [5], [6], [7]. The number of simulations by finite element method will vary depending on the number of present electrical conductors in the cable. Each simulation will provide an energy value that will allow us to determine the lumped parameter (capacity, resistance and inductance) matrices. In addition, these simulations will be performed for several frequencies to capture the evolution of the skin and proximity effects. The design model is carried out by Salome platform, the computation by *code_Carmel3D* (co developed by laboratory L2EP and EDF R&D) [8], [9].

Once the lumped parameter matrices are obtained, the modal decomposition method is used to extend the model of cable to higher frequencies. To validate the proposed method a comparison is made between simulation results and measurements extracted from [10]. A particular attention is paid on evolution according to frequency of the impedance of the cable in open-circuit and short-circuit operation modes.

II. METHODS

In this section, the formulations used to calculate the lumped parameters are introduced. Based on energy method, the lumped parameters are obtained from the finite element model. It is also shown how impedances matrices are obtained from all performed simulations. The Transmission Line Method (TLM) and modal decomposition are also briefly presented. Finally, the method used to reduce matrices [R] [L] [C] is discussed, by presenting the matrix of connections that allow calculating impedances in the same configurations (short circuit, open circuit, common mode and differential mode) as cable impedance measurement

A. Formulations and Finite Element Method

In this study, the value of the capacitance matrix is supposed to be not frequency dependent. Thus, the capacitance between the wires is calculated using the electric scalar potential formulation. However, for the resistance and inductance matrices which vary with the frequency, the two magnetoharmonic potential formulations are used.

Electrostatic case

To determine the capacitance between the conductors we must be solve an electrostatic problem free of space charges. The computation can be carried out with the electric scalar potential φ formulation which can be written as

$$\text{div}\varepsilon\mathbf{grad}\varphi_I = -\text{div}\varepsilon\mathbf{grad}\varphi_S \qquad (1)$$

with ε the electric permittivity and φ_I, φ_S the electric scalar potential unknown and source. The boundary condition $\mathbf{E}\times\mathbf{n}=0$ on the border Γ_E is prescribed by imposing $(\mathbf{grad}\varphi_S) \times n = 0$ and $(\mathbf{grad}\varphi_I) \times n = 0$ on the border Γ_E.
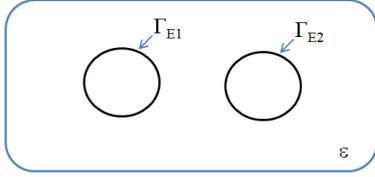
Figure 1.   Studied domain of electrostatic case

Magnetoharmonic case

As mentioned before, the objective is to determine the resistance and inductance matrices which depend on the skin effect and proximity effects.   After having determined the capacitance matrix of the cables made of a set of wires, the resistance and inductance matrices are calculated in function of the frequency by solving the magnetoharmonic formulations. Several formulations in terms of fields or potentials can be used. In our case the potential formulations A-φ and T-Ω are used in order to take into account naturally the coupling between the formulations adopted to model the conductive and non conductive parts.

The magnetic vector potential $\mathbf{A}$ and the electric scalar potential φ are defined such that the magnetic field $\mathbf{B}$ and vector $\mathbf{A}$ are related by $\mathbf{B}=\mathbf{curl\ A}$ and the electric field E is equal to $\mathbf{E}=-j\omega\mathbf{A}-\mathbf{grad}\ \varphi$. Combining the previous equations with the Ampere law ($\mathbf{curl\ H = J}$, $\mathbf{H}$ the magnetic field and $\mathbf{J}$ the current density) and the behavior laws ($\mathbf{B}=\mu\mathbf{H}$ and $\mathbf{J}=\sigma\mathbf{E}$ with μ the permeability and σ the conductivity), the partial derivative equation to solve is:

$$\mathbf{curl}\frac{1}{\mu}\mathbf{curlA} = \mathbf{J_S} - \sigma(j\omega\mathbf{A}+\mathbf{grad}\,\varphi) \qquad (2)$$

where $\mathbf{J_S}$ is the current density like such account the source term.

The boundary conditions prescribed on $\mathbf{B}$ $(\mathbf{B.n=0})$ and $\mathbf{E}$ $(\mathbf{E}\times\mathbf{n=0})$ are imposed applying $\mathbf{A}\times\mathbf{n}=0$ on Γ_B and $\mathbf{A}\times\mathbf{n}=0$ and $\varphi=0$ on Γ_E respectively.

Another potential formulation can be introduced, the electric vector potential formulation $\mathbf{T}$ and the magnetic scalar potential formulation Ω are introduced such that:

$$\mathbf{J} = \mathbf{J_S} + \mathbf{J_{ind}} = \mathbf{curlT_S} + \mathbf{curlT} \qquad (3)$$

where $\mathbf{T_S}$ is defined such that $\mathbf{J_S}=\mathbf{curl\ T_S}$ and the unknown term $\mathbf{J_{ind}}=\mathbf{curl\ T}$.

Consequently the equation to solve on the conductive part is given by:

$$\mathbf{curl}\frac{1}{\sigma}(\mathbf{curlT_S} + \mathbf{curlT}) = -j\omega\mu(\mathbf{T_S} + \mathbf{T} - \mathbf{grad}\Omega) \qquad (4)$$

The boundary conditions of type $\mathbf{J}$ and $\mathbf{H}$ on the boundary Γ_H by imposing $\mathbf{T}\times\mathbf{n}=0$ on and $\Omega=0$ on Γ_H.

The main interest to solve both formulations is to obtain two values of lumped parameters (one for each formulation). It is expected that obtained results will be close and the numerical errors introduced by the Finite Element method will be small. If it is not the case, the mesh should be refined.

B.   Determination of impedance matrices

- Balance of energy – calculating C

To explain the procedure for the determination of the capacitance matrix, the simplest case of non-shielded cable is presented (see Figure 2.). On this type of cable, there are 3 values of capacitance: one between wires and the other two connecting each wire to the ground. Three simulations will be performed to determine these three capacities.
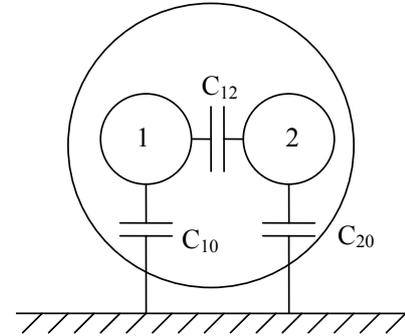
Figure 2.   Capacitances of two wires unshielded cable

The electrical potential of the ground is still imposed to zero. The electrical potential of the conductors 1 and 2 are respectively denoted V_1 and V_2. Three configurations for the simulations are presented in table 1 and the expressions of the electrostatic energy potentials in function of the capacitances are given.

TABLE I: CONFIGURATION OF THE SIMULATIONS

Configuration	V_1 (V)	V_2 (V)	W(J)
1	1	0	$\frac{1}{2}(C_{10}+C_{12})V_1^2 = \frac{1}{2}C_{11}V_1^2$
2	0	1	$\frac{1}{2}(C_{20}+C_{12})V_2^2 = \frac{1}{2}C_{22}V_2^2$
3	1	2	$\frac{1}{2}(C_{10}V_1^2 + C_{20}V_2^2 + C_{12}(V_1-V_2)^2)$

According to the equations given in the Table I, the capacitances C_{10}, $C_{20,}$ C_{12} can be calculated from the values of the energy determined in the 3 configurations by solving a system of equations.   In the case of a cable with 3 wires, 6 capacitances have to be calculated and therefore, 6 simulations to carry out. It can be noted that the self capacitance is the sum

of two capacities. Finally, for a general case, we can write capacity matrix [C] as:

$$C = \begin{bmatrix} C_{11} & -C_{12} & -C_{13} & \dots & -C_{1n} \\ -C_{21} & C_{22} & -C_{23} & \dots & -C_{2n} \\ -C_{31} & -C_{32} & C_{33} & \dots & -C_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ -C_{n1} & -C_{n2} & -C_{n3} & \dots & C_{nn} \end{bmatrix}$$

(5)

- Calculating R, L

Considering again the two wires cable of the figure 1 and the same approach based on the calculation of the energy (Joule losses in the case of the resistance), the values of R and L matrices can be found. In general, if the wires are flown by an electric current, the Joule losses and the magnetic energy are expressed as follows:

$$\begin{cases} P_{Joules} = I_1^2.R_{11} + I_2^2.R_{22} + 2I_1I_2R_{12} \\ W_{mag} = \frac{1}{2}.L_{11}I_1^2 + \frac{1}{2}.L_{22}I_2^2 + L_{12}I_1I_2 \end{cases}$$

(6)

where R_{11}, R_{22}, L_{11}, L_{22} are respectively the self resistances and inductances; and R_{12}, L_{12} are the mutual terms.

To determine the couple of terms (R_{11}, L_{11}) and (R_{22}; L_{22}) it can be canceled respectively either I_1 or I_2. Finally (R_{12}, L_{12}) are obtained by prescribing I_1 to 1 A and I_2 to -1A. To take into account the evolution of the resistance according to the skin effect and the proximity effect, simulations must be carried out at several frequency values. It should be noted that self resistance values corresponds to Joule losses in the two wires when only one is supplied. Indeed, although the current I_2 is zero (corresponding to an open circuit) the current density induced by proximity effect in the wire 2 is not zero and, as a result, generates losses in this wire.

C. Transmission Line Model and Analysis Modal Approach

In the previous sections, FEM was used to calculate R, L and C, or in other words, the impedance Z and the admittance Y of cable, according to frequency. In this section, the TLM method combined with modal decomposition is used to calculate the impedance of the cables in order to determine the resonance frequency of the cable. The steps of calculation are described in [1], [5], [6].
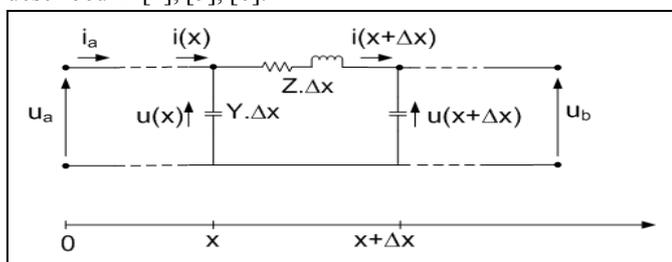
Figure 3.   Analysis of a small portion Δx of one transmission line

- Transmission Line Matrix Method

Generally, two parallel long transmission lines are considered to introduce the method, a conductor wire and a reference wire.

In the Figure 3. , U_a is the applied voltage at the input of the line; U_b is the output voltage at the end of the line. The equations are easily obtained starting with the relations between the currents and the voltages on a small portion Δx of the line.

$$\begin{cases} \dfrac{du(x)}{dx} = -Z.I(x) \\ \dfrac{di(x)}{dx} = -Y.U(x) \end{cases}$$

(7)

The impedance Z and the admittance Y are found from the matrices R, L, C. To decouple the equations of the system (7), a modal decomposition must be carried out. The principle of modal analysis is the use of orthogonal matrices Q_u and Q_i suitable for transforming the real voltages and currents u, i to modal voltages and currents u_m, i_m. The modal voltages and currents are obtained by:

$$\begin{cases} u_m = Q_u^{-1}u \\ i_m = Q_i^{-1}i \end{cases}$$

(8)

The impedance and admittance matrices Z_m and Y_m become diagonal:

$$\begin{cases} Z_m = Q_u^{-1}.Z.Q_i \\ Y_m = Q_i^{-1}.Y.Q_u \end{cases}$$

(9)

To calculate the matrices Q_u and Q_i, it is necessary to solve the eigenvalues problem defined by the following equations:

$$\begin{cases} (YZ).Q_i = Q_i.(Y_m.Z_m) \\ (ZY).Q_u = Q_u.(Z_m.Y_m) \end{cases}$$

(10)

The system of equation (7) can be written under a modal form:

$$\begin{cases} \dfrac{du_m(x)}{dx} = -Z_m.i_m(x) \\ \dfrac{di_m(x)}{dx} = -Y_m.u_m(x) \end{cases}$$

(11)

Each mode can be calculated independently. As a result, it can obtained a propagation constant and a characteristic impedance by mode that are denoted respectively $\beta_{m,k}$ and $Z_{ch,k}$ with k index corresponding to the k-th mode. Thus, a propagation matrix can be expressed for each mode .

$$\begin{bmatrix} u_{m,k}(l) \\ i_{m,k}(l) \end{bmatrix} = \begin{bmatrix} \cosh(\beta_{m,k}l) & -Z_{ch,k}\sinh(\beta_{m,k}l) \\ -1/Z_{ch,k}\sinh(\beta_{m,k}l) & \cosh(\beta_{m,k}l) \end{bmatrix} \begin{bmatrix} u_{m,k}(0) \\ i_{m,k}(0) \end{bmatrix} \quad (12)$$

with l the conductor length. In the following, the propagation matrix A_m is introduced. In order to calculate the real currents and voltages, propagation matrix can be written:

$$\begin{bmatrix} u^{out} \\ i^{out} \end{bmatrix} = Q_{ui}A_mQ_{ui}^{-1}\begin{bmatrix} u^{in} \\ i^{in} \end{bmatrix} = A\begin{bmatrix} u^{in} \\ i^{in} \end{bmatrix} \quad (13)$$

with the transformation matrix $Q_{ui} = \text{diag}\{Q_u, Q_i\}$.

- Connection Matrix

The matrix A provides a link between the input currents and voltages u^{in} and i^{in} and output voltages and currents u^{out} and i^{out}. To impose a unique solution, the connections between the cables should be taken into account. Consequently, these connections can be represented by equations linking currents and voltages which are added to the system (13).

Take again, the two-wire cable of the Figure 4.
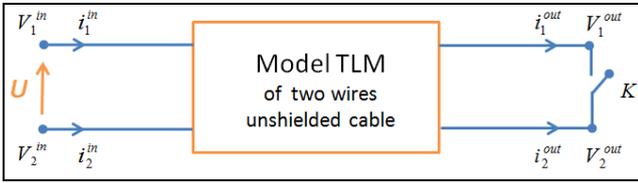
Figure 4. Equivalent circuit of unshielded cable

The vectors of the input quantities X^{in} and the vector of quantities output X^{out} are defined such that:

$$X^{in} = \begin{pmatrix} V_1^{in} \\ i_1^{in} \\ V_2^{in} \\ i_2^{in} \end{pmatrix} \text{ and } X^{out} = \begin{pmatrix} V_1^{out} \\ i_1^{out} \\ V_2^{out} \\ i_2^{out} \end{pmatrix}$$

where $X^{out} = AX^{in}$.

The source vectors S^{in}, S^{out} corresponds to imposed initial conditions by the external circuit. In our case, only the voltage will be imposed on the input of the wire 1according to:

$$S^{in} = \begin{pmatrix} U \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } S^{out} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

It remains now to determine the connection matrix between the input wires and the output wires. The matrices C^{in}, C^{out} will be determined based on the relation between current and voltage.

$$\begin{pmatrix} C^{in} & 0 \\ 0 & C^{out} \end{pmatrix}\begin{pmatrix} X^{in} \\ X^{out} \end{pmatrix} = \begin{pmatrix} S^{in} \\ S^{out} \end{pmatrix} \quad (14)$$

C^{in} is determined from the relation between the current and voltages of the two input wires:

$$\begin{cases} V_1^{in} = U \\ V_2^{in} = 0 \end{cases} \Rightarrow C^{in} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

C^{out} is calculated based on the conditions of connection at the extremity of the cable then, two modes are considered:

- Short circuit (K close) :

$$\begin{cases} V_1^{out} = V_2^{out} \\ i_1^{out} + i_2^{out} = 0 \end{cases} \Rightarrow C^{out} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (16)$$

- Open circuit (K open) :

$$i_1^{out} = i_2^{out} = 0 \Rightarrow C^{out} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (17)$$

Finally, the system to solve is not square but the solution is easily obtained by multiplying the system by its own transposed.

$$\begin{pmatrix} C^{in} \\ C^{out}A \end{pmatrix}X^{in} = \begin{pmatrix} S^{in} \\ S^{out} \end{pmatrix} = BX^{in} = S \Rightarrow X^{in} = (B^tB)^{-1}B^tS \quad (18)$$

III. VALIDATIONS AND APPLICATIONS

A. – Two wires unshielded cable

Radius of conductor	r_1 (mm)	0,69
Radius of insulation PVC	r_2 (mm)	1,44
Distance between centres of two conductors	D (mm)	2,88
Radius of rubber	r_3 (mm)	4,35
Conductivity of conductor	σ_{cond} (MS/m)	45.94
Relative permittivity of PVC and rubber	ε_r	2,3
Relative permeability	μ_r	1
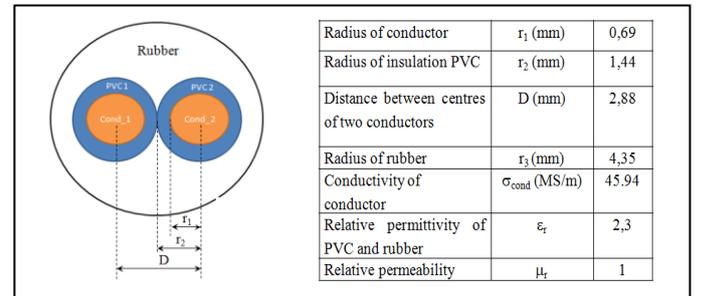
Figure 5. Configuration of the two wires unshielded cable

Characteristic cable

The cable is composed of two conductors coated with PVC, and the whole is placed in a rubber sheath. The geometrical characteristics and electrical parameters of the cable are presented in the Figure 5.

Modeling of High Frequency (HF) behavior

As the cable consists of two conductors, the electrical characteristics of this cable are presented by matrices R, L, and C of size 2×2.
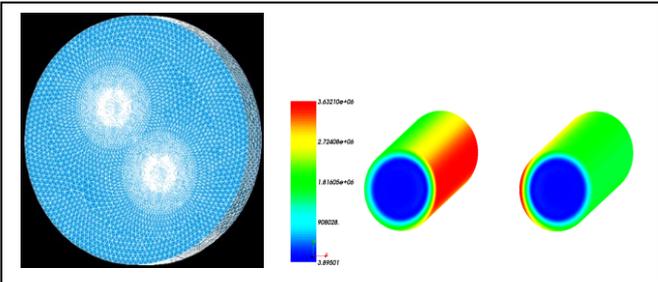
Figure 6.   Mesh and distribution of current at f = 500 kHz

Figure 6. shows the mesh of this cable obtained with Salome Platform and *code_Carmel3D* and the distribution of the eddy current when two currents are imposed at 500 kHz in an opposite direction. The behavior HF of cable like the skin and proximity effects is modeled well.

It can be noted that the mesh will be the same for all frequency range calculations. Therefore, attention is paid on the FE modeling of conductor domains in order to obtain a good representation of skin and proximity effects.
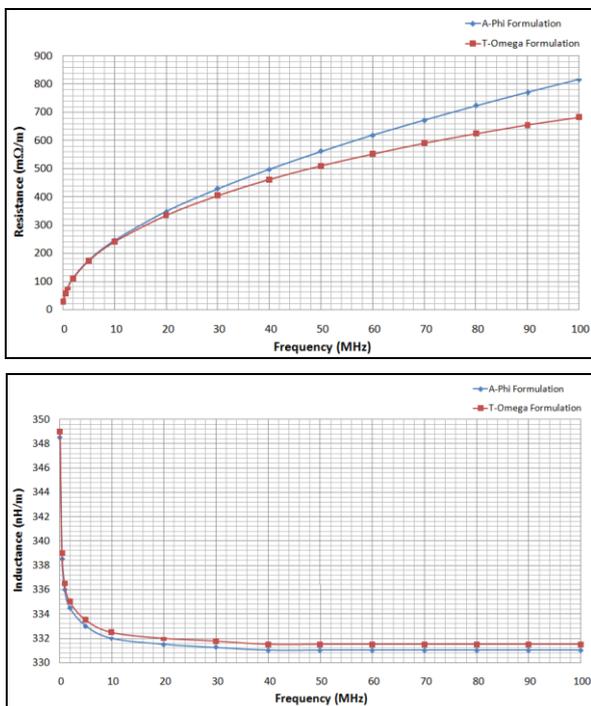
Figure 7.   Evolutions of R, L of one wire according to frequency by two formulations

Solving the problem magnetoharmonic by finite element method, values of resistance and inductance are obtained according to frequency. The computation is carried out for only 15 values of frequency. The results are obtained by using *code_Carmel3D* (two formulations). From Figure 7. , it can be seen that the results found by the computational approaches are quite similar. The difference is very small at 500kHz, and around 2% at 10MHz. The evolutions of R_{11}, L_{11} according to frequency are obtained by interpolation method.

The gap between these two formulations (resistive term) is higher increasing the frequency beyond 10MHz.

Impedance computation

From the matrices [R], [L] according to frequency and the matrix [C] the short-circuit and the open circuit impedance can be calculated by applying the method detailed in the previous section.

It can be found that the resonance frequency of the open circuit mode corresponds to the anti-resonance frequency of the short circuit mode and vice versa.

The results of this cable 1m length for a range [0.1 Hz, 100 MHz] in both short circuit and open circuit modes are shown in Figure 8.   In this frequency range, two resonance frequencies are identified at 49 MHz and 98.5 MHz. A comparison between the simulation and the measurement results (extracted from [10]) is presented in table II. It can be observed that these results are in good agreement.

Moreover with this approach it is easy to carry out a computation for another cable length. Using the same approach, the results for a cable of 10 m are shown in Figure 9.   and the comparison with the experimental results is presented in Table III.
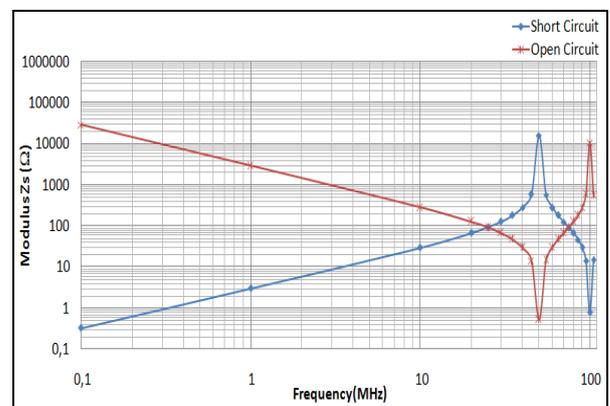
Figure 8.   Impedance Modulus of the two wires unshielded cable 1m in length at SC and OC

TABLE II.   COMPARISON BETWEEN THE VALUES OF MEASURED AND CALCULATED RESONANT FREQUENCIES FOR A CABLE OF 1 m

Resonance frequency (MHz)	Short Circuit		Open Circuit	
	f_{01}	f_{02}	f_{01}	f_{02}
Simulation	49.0	97.8	49.0	98.5
Measurements	44.2	86.1	44.3	92.3
Difference (%)	9.79	11.96	9.59	6.29
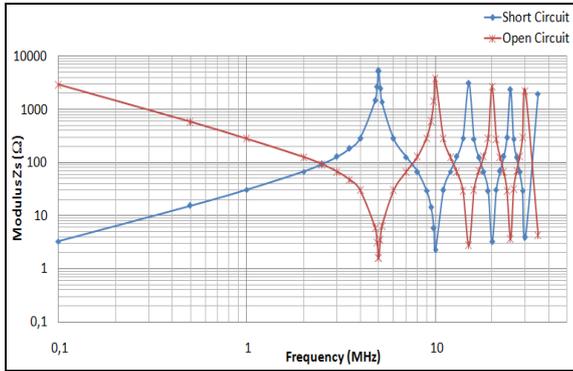
Figure 9.   Impedance Modulus of the two wires unshielded cable of 10m length at SC and OC.

TABLE III.   COMPARISON BETWEEN THE VALUES OF MEASURED AND CALCULATED RESONANT FREQUENCIES FOR A CABLE OF 10 m

Resonance frequency (MHz)	Short Circuit		Open Circuit	
	f_{01}	f_{02}	f_{01}	f_{02}
Simulation	4.91	9.89	4.91	9.90
Measurements	3.95	7.96	3.95	7.96
Difference (%)	19.55	19.51	19.55	19.59

B.   Two wires shielded cable

Characteristic cable

To reduce the ElectroMagnetic Compatibility (EMC) impact of this cable, a shielding layer of 0.2 mm thickness has been added. The shielded material is the same as the one of the wires (σ= 45.94 MS/m). The radius of conductor: 0.5 mm, of PVC coated the conductor: 1.25 mm and of external PVC: 2.9 mm.

Figure 10. (on the right) depicts the distribution of the eddy currents when two currents are imposed through the wires at 500 kHz in an opposite direction. The behavior HF is presented well as the skin and the proximity effects appear in the cable.
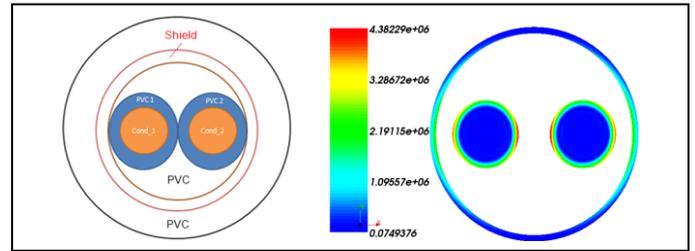
Figure 10. Characteristic of the two wires unshielded cable distribution of current at f = 500 kHz.

Modeling of HF behavior

As the cable consists of two conductors with a shielded layer then, the electrical characteristics of this cable are modeled using matrices R, L, C of size 3×3.
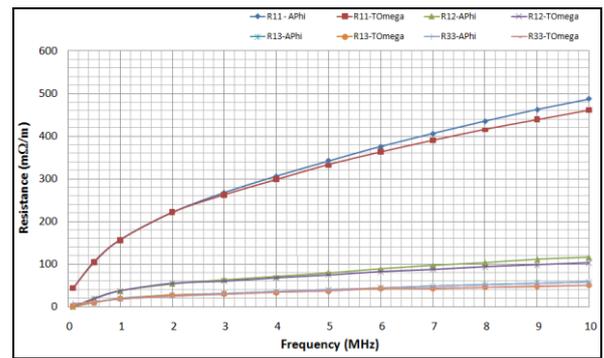
Figure 11. Evolution of R according to frequency
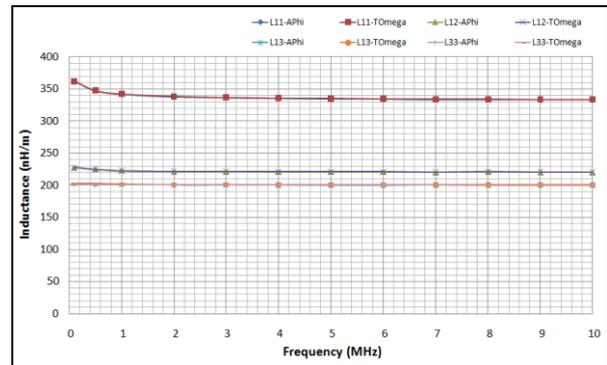
Figure 12. Evolution of L according to frequency

Figure 11. and Figure 12. shows the evolution of the coefficients of the resistance and inductance matrices of the cable in a range [0.1; 10] MHz. The four self values of conductor (R_{11}, L_{11}) (R_{22}, L_{22}) and of the shield (R_{33}, L_{33}), mutual values between the two conductors (R_{12}, L_{12}) and between one conductor and the shield (R_{13}, L_{13}) are presented. It can be seen that the variation of the resistances is much more important than the variation of the inductances as expected by the theory for this kind of cable.

On the other hand, solving the electrostatic problem by finite element method, the capacitance: C_{12} between two conductors, C_{10}, C_{20} between the conductors and shield (table

IV) are obtained. In this study, the capacitances are assumed constant in function of frequency.

TABLE IV. VALUES OF CAPACITANCES OF THE TWO-WIRES SHIELDED CABLE

Capacity	Value
C_{12} (pF/m)	17,4
C_{10}, C_{20} (pF/m)	136

Impedance computation

With this cable, two configuration modes can be considered: Common Mode and Differential Mode. For each mode, two configurations of circuit are studied: Short Circuit and Open Circuit.

Common mode
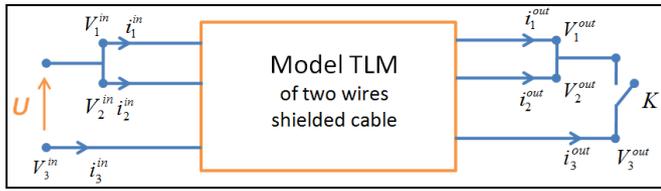
Figure 13. Equivalent circuit of common mode transmission.

In common mode, the current flows by the tow wires and returns through the shield (Figure 13.).

General matrix equation to solve (10) with the input unknowns, output unknowns and the sources:

$$S^{in} = \begin{pmatrix} U \\ 0 \\ U \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

C^{in} is determined from the relation of current, voltage at the input of the line:

$$\begin{cases} V_1^{in} = V_2^{in} = U \\ V_3^{in} = 0 \end{cases} \Rightarrow C^{in} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

C^{out} is calculated based on the conditions at the end of the cable, then two cases are considered:

- Short circuit (K close)

$$\begin{cases} V_1^{out} = V_2^{out} = V_3^{out} \\ i_1^{out} + i_2^{out} + i_3^{out} = 0 \end{cases} \Rightarrow C^{out} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

- Open circuit (K open)

$$i_1^{out} = i_2^{out} = i_3^{out} = 0 \Rightarrow C^{out} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

From these matrices, equation (18) is solved. The results of this cable 10m length for a range [0.1; 10] MHz in both short circuit (SC) and open circuit (OC) are shown in Figure 14. In this frequency range two resonance frequencies are obtained: 3.77 MHz and 7.60 MHz for SC; 3.79 MHz and 7.61 MHz for OC.
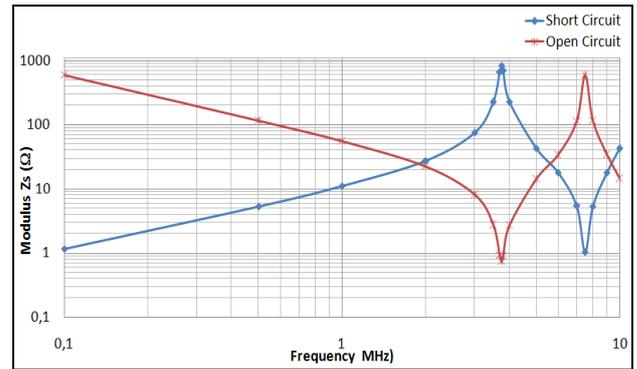
Figure 14. Modulus of impedance according to the frequency of the two wires shielded cable of 10m length – common mode - at SC and OC.

In the table V the simulation and measurements results are compared. For this shielded cable, the results are always close.

TABLE V. COMPARISON BETWEEN THE VALUES OF MEASURED AND CALCULATED RESONANT FREQUENCY FOR A CABLE OF 10 m

Resonance frequency (MHz)	Short Circuit		Open Circuit	
	f_{01}	f_{02}	f_{01}	f_{02}
Simulation	3.89	7.78	3.89	7.78
Measurements	4.40	9.14	4.40	8.85
Difference (%)	11.59	14.88	11.59	12.09

Differential mode

In this mode, the current flows through one wire and returns through the other. In this configuration, the shield doesn't connect to the wires (Figure 15)
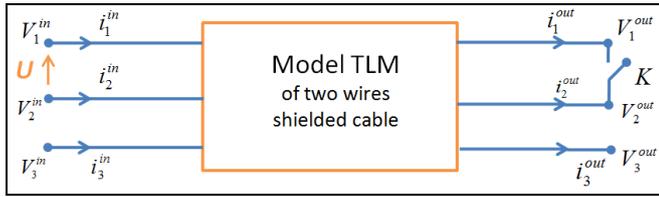
Figure 15. Equivalent circuit of differential mode transmission.

The unknowns related to the input and output of the cables are the same as common mode case. However, the applied source matrix in this case is changed:

$$S^{in} = \begin{pmatrix} \dfrac{U}{2} \\ 0 \\ \dfrac{-U}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Based on the relation between the currents and the voltages at the input of the cable, it can be written:

$$\begin{cases} V_1^{in} = \dfrac{U}{2} \\ V_2^{in} = \dfrac{-U}{2} \\ V_3^{in} = 0 \end{cases} \Rightarrow C^{in} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

C^{out} is calculated according to the connections at the end of the cables, so two cases are considered:

- Short circuit (K close) :

$$\begin{cases} V_1^{out} = V_2^{out} \\ i_1^{out} + i_2^{out} = 0 \\ i_3^{out} = 0 \end{cases} \Rightarrow C^{out} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Open circuit (K open) :

$$i_1^{out} = i_2^{out} = i_3^{out} = 0 \Rightarrow C^{out} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The results for a 10m length cable obtained for a frequency range [0.1; 10] MHz in both short circuit and open circuit configurations are shown in Figure 16. In this area, there are two resonance frequencies: 3.88 MHz and 7.75 MHz for SC; 3.89 MHz and 7.76 MHz for OC.
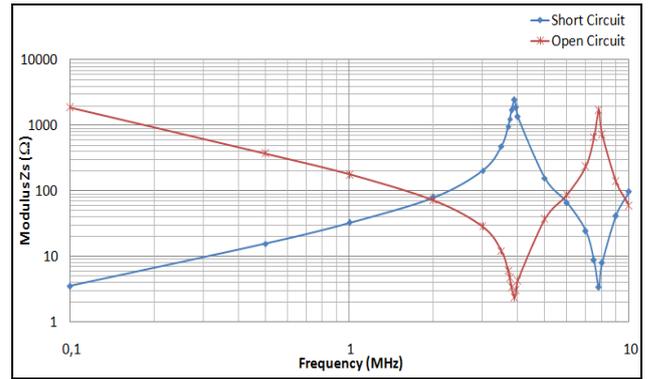
Figure 16. Modulus of impedance according to the frequency of the two-wires shielded cable of 10m length – differential mode - at SC and OC

It should be noted that the resonance frequencies obtained in differential mode are not the same as the resonance frequencies obtained in common mode. From table VI it can be seen that measurements and the simulation results agree well.

TABLE VI. COMPARISON BETWEEN THE VALUES OF MEASURED AND CALCULATED RESONANT FREQUENCY FOR THE TWO-WIRES SHIELDED CABLE OF 10m IN DIFFERENTIAL MODE

Resonance frequency (MHz)	Short Circuit		Open Circuit	
	f_{01}	f_{02}	f_{01}	f_{02}
Simulation	3.92	7.87	3.92	7.87
Measurements	4.31	8.83	4.25	8.83
Difference (%)	9.05	10.87	7.76	10.87

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IV. DISCUSSION AND CONCLUSIONS

This paper presents a method applied to determine the impedances of cables according to the frequency. This method presents two advantages. First, the modeling of proximity effect in high frequency is well done. Second, the connection matrix is introduced. The impedance of other configuration of system according to frequency can be determined by changing this matrix. The calculations have been performed on two different cables: shielded and unshielded cable. In addition, on

the shielded cable, computations have been carried out taking into account the two operation modes: common mode and differential mode. Comparison of simulations with the measurements on the value of the frequency of resonance showed that the method gives good results. Moreover, this method can be applied to calculate the resonant frequency for other systems such that the three phase cable in the grid, the four wire HVDC cable. The presented models will be introduced in software EMTP.

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