Frequency Dependant Current Distribution and Resistance Coefficient of Aluminium Conductors

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***Abstract*—Aluminium conductors, such as ACSR (Aluminium Conductor Steel Reinforced) or ACCR (Aluminium Conductor Composite Reinforced) are widely applied in electric power transmission and distribution systems. For that reason, we decided to investigate the behavior of those conductors, making an accent on current distribution inside them and resistance coefficient determination. In this paper we do present all calculation results for two ACSR and two ACCR, of similar characteristics, at frequencies up to 2500Hz. Obtained results, presented graphically, confirmed our expectations. The entire calculation was carried out applying COMSOL Multiphysics 3.5a computer program package.**

Keywords-Current distribution; graphical presentation; inhomogeneous conductors; resistance coefficient

I. Introduction

In all time-varying electromagnetic fields, inside conductors and inside all neighboring conducting bodies, skin effect and proximity effects appear. The consequences of those effects are non-uniform current distribution across conductor's cross-section, increased Joule's losses and virtual increasing of conductor's resistance. Non-uniform current distribution and increased conductor's resistance can be presented by several functional dependences. Current distribution is usually presented as dependence of current density vector magnitude on the distance from the conductor's axes. Two of the most common presentations of resistance coefficient, $k_R = R'_{\approx} / R'_{=}$, are the dependence on frequency and on factor $\sqrt{f / R'_{=}}$, where $R'_{\approx}$ denotes the AC resistance per kilometer of conductor's length, $R'_{=}$ is the DC resistance per kilometer of conductor's length and f is the applied frequency. Calculated resistance coefficients for homogenous conductors of different cross-sections, together with both mentioned graphical presentations were shown in [1]-[3]. Special accent on the second resistance coefficient functional dependence and its advantages was given in [1]. It was shown in [1] that, for significant skin effect in any homogenous conductor, resistance coefficient depends linearly on $\sqrt{f / R'_{=}}$ term,

$$\frac{R'_{\approx}}{R'_{=}} = \sqrt{\frac{\mu}{4}} \sqrt{\frac{f}{R'_{=}}} .$$

On the other hand, both aluminium conductors, ACSR (Aluminium Conductor Steel Reinforced) and ACCR (Aluminium Conductor Composite Reinforced) are not homogenous materials. The typical cross-section of both conductors is given in Fig. 1, [4]-[6].

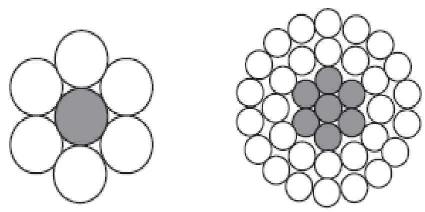

Figure 1. Typical cross-section of an ACSR or an ACCR conductor

Due to complex cross-section geometry, shown in Fig. 1, and even more due to different conductivities of supporting core (steel or aluminium oxide composite) and conductive aluminium wires around the core, we do expect much more complicated relations. Moreover, the nonlinear ferromagnetic core of ACSR, is expecting to play a significant role in all calculations and obtained results.

Investigating the possibility of the conductor's geometry simplification, a simplified model, shown in Fig. 2 b), is adopted for all calculations.

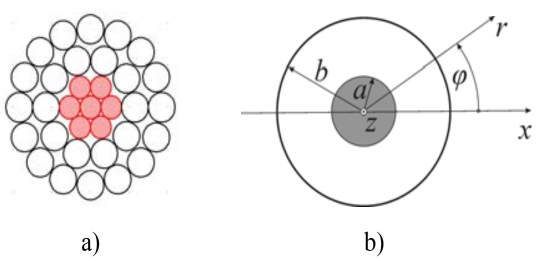

Figure 2. a) Typical cross-section of an ACSR or an ACCR conductor, and b) and simplified model

It is shown that the results of calculations on simplified model correspond to the results of the real conductor's investigations [3], [7]. For that reason only the results obtained on simplified model will be presented in this paper.

II. Theoretical Approach

In order to determine conductors' AC resistance, total current distribution, induced electric field, and Joule's losses power must be calculated.

Considering typical cross-section shown in Fig. 2 b), the problem can be presented in cylindrical coordinate system with the conductor's axes positioned in z-axes of chosen coordinate system. The problem can also be considered as two-dimensional and linear. Although ACSR steel core is made of nonlinear, ferromagnetic material, magnetic field inside the core is negligible, so the permeability dependence on magnetic flux density magnitude is negligible as well.

Consequently, current distribution and resistance coefficient calculation can be performed in complex, two-dimensional domain, applying complex magnetic vector potential, which has only z-component and is a solution of following partial differential equation [8], [9],

$$\Delta \underline{A}_z - j\omega\mu\sigma\underline{A}_z = -\mu\underline{J}_z \,. \tag{1}$$

In cylindrical coordinate system, the above equation can be written as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \underline{A}_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \underline{A}_z}{\partial \varphi^2} - j\omega\mu\sigma\underline{A}_z = -\mu\underline{J}_z \,. \tag{2}$$

The calculated values of complex magnetic vector potential define the complex induced electric field strength vector and the complex induced current density vector, both in z-direction,

$$\vec{\underline{E}}_{ind} = -j\omega\vec{\underline{A}} \,, \qquad \vec{\underline{J}}_{ind} = -j\omega\sigma\vec{\underline{A}} \,. \tag{3}$$

Total complex current density vector is the sum of imposed complex current density vector, $\vec{\underline{J}}_0$ and induced current density vector, $\vec{\underline{J}}_{ind}$,

$$\vec{\underline{J}}_{tot} = \vec{\underline{J}}_0 + \vec{\underline{J}}_{ind} \,. \tag{4}$$

Joule' losses power per kilometer of conductor's length can be determined by integration across the conductor's cross-section,

$$P'_J = \int_{Scs} \frac{J_{tot}^2}{\sigma} dS \,. \tag{5}$$

AC resistance per kilometer of conductor's length can now be calculated as,

$$R'_{\approx} = \frac{P'_J}{\left|\underline{I}_0\right|^2} \,. \tag{6}$$

In order to define the resistance coefficient,

$$k_R = \frac{R'_{\approx}}{R'_{=}} \,, \tag{7}$$

the DC resistance must be determine as well. This resistance can be calculated applying the well known formulae for parallel connection of defined number of wires, N_k [3],

$$R'_{=core} = \frac{1}{\sigma_{core}N_{core}S_{core}} \,, \qquad R'_{=Al} = \frac{1}{\sigma_{Al}N_{Al}S_{Al}} \,. \tag{8}$$

$$R'_{=} = \frac{R'_{=core}\cdot R'_{=Al}}{R'_{=core} + R'_{=Al}} \,. \tag{9}$$

III. Model

DC resistances were calculated applying (8) and (9). In ACSR case the conductivities, $\sigma_{core} = \sigma_{Fe}$ and σ_{Al} were measured [2] and those values, applied in all resistance calculations are,

$\sigma_{Al} = 3.55\cdot 10^7$ S/m, $\qquad \sigma_{Fe} = 0.559\cdot 10^7$ S/m.

In an ACCR, aluminium and composite aluminium oxide conductivities are taken from [6],

$\sigma_{Al} = 3.48\cdot 10^7$ S/m, $\qquad \sigma_{AlO} = 1.405\cdot 10^7$ S/m.

AC resistance calculations were carried out applying AC/DC module of COMSOL Multiphysics 3.5a computer program package [10]. A 2D model, representing a cross-section of conductor in surrounding air domain, is set up. The mode "Quasi-static, Magnetic/Perpendicular Induction Currents/Vector Potential" is chosen, together with "Time-harmonic analysis".

As shown in [3], for resistance calculations the simplified model, shown in Fig. 2 b), together with cylindrical coordinate system, can be satisfactory applied.

In all calculating models, rms value of current is chosen to be the same in every conductor, equal, I = 1A. Having a linear problem, this value will not influence the final results. All conductors are surrounded by air, with defined electromagnetic characteristics, ε_r=1, μ_r=1, σ=0.

In order to define boundary conditions, an exterior boundary was involved. This boundary was supposed to be the circle of radius R_b=1m, out of which there is no electromagnetic field. [2], [3]. On the boundary, the surface current density,

$$J_s = -\frac{I}{2\pi R_b} \tag{10}$$

was supposed, providing the same return current in order to ensure zero electric and magnetic field outside the chosen modeling domain.

Magnetic vector potential is continuous across all interior boundaries between all inner conductors and between external aluminium conductors and surrounding air.

The applied software offers extended post processing calculations, so the resistance coefficient could be calculated easily.

All calculation results will be commented in following chapters. Moreover, the calculation results for two types of conductors, ACSR and ACCR, will be compared and commented as well.

IV. Obtained Results

In order to enable comparison between two different type of conductors, we chose to investigate three pairs of conductors with similar cross-sections. According to [4] and [6], the conductors with similar cross-sections are: ACSR 240/40mm^2 and ACCR Hawk 477, ACSR 150/25mm^2 and ACCR Ostrich 300, as well as ACSR 680/85mm^2 and ACCR Martin 1351. Some constructive characteristics of both conductors' types are given in Table I.

TABLE I. Constructive Elements of ACSR and ACCR with Similar Cross-Section

Conductor type	**Aluminium outer wires**		Core
	No. & diameter of wires	***No. of layers***	***No. & diameter of wires***
	n x mm		***n x mm***
ACSR 240/40 mm^2	26(10+16)×3.45	2	7(1+6)×2,68
ACCR Hawk 477	26(10+16)×3.40	2	7(1+6)×2,70
ACSR 150/25 mm^2	26(10+16)×2.7	2	7(1+6)×2.1
ACCR Ostrich 300	26(10+16)×2.7	2	7(1+6)×2.1
ACSR 680/85 mm^2	54(10+16+28)×4.0	3	19(1+6+12)×2.4
ACCR Martin 1351	54(10+16+28)×4.0	3	19(1+6+12)×2.4

As an example, the current distribution inside the ACSR 240/40 mm^2 at 50 Hz is given in Fig. 3 and the current distribution at 450 Hz is given in Fig. 4.

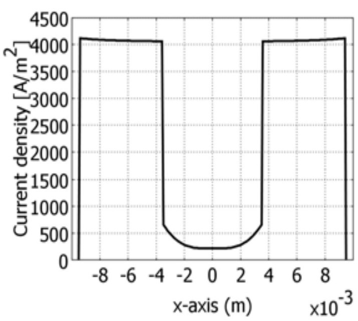

Figure 3. Current distribution across the ACSR 240/40 mm^2, at 50 Hz

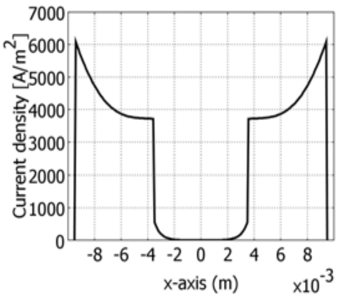

Figure 4. Current distribution across the ACSR 240/40 mm^2, at 450 Hz

Two basic characteristics in ACSR behavior could be observed in Fig 3. First, due to different conductivity values, even at 50 Hz a very small amount of current exists in the steel core. This amount is additionally smaller due to significant skin effect, produced by big permeability of steel part of the conductor. At 50Hz skin effect inside aluminium part is not significant. Hence, current distribution in this part is almost uniform.

At the frequency of 450 Hz the skin effect is much more emphasized, which can be noticed in Fig. 4. First, there is no current inside the steel core and the current distribution inside aluminium part is no longer uniform; the current density vector magnitude increases toward conductor's surface.

The same diagrams for the similar ACCR, Hawk 477, are presented in Fig. 5 and Fig. 6.

In Fig. 5 the main characteristics of ACCR can be seen. The difference in conductivity values between the core and the external conducting wires are much smaller comparing ACSR. Consequently, inside the core much bigger percentage of current exists at all frequencies.

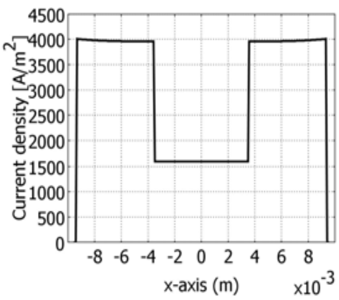

Figure 5. Current distribution across the ACCR Hawk 477, at 50 Hz

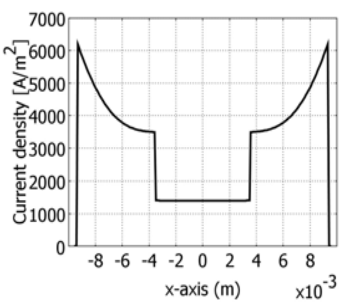

Figure 6. Current distribution across the ACCR Hawk 477, at 450 Hz

Besides, the relative permeability of aluminium oxide composite core is equal to one, so the skin effect is almost negligible at 50 Hz.

In Fig. 6 a significant skin effect can be noticed, presented by smaller amount of current inside the core and by non-uniform current distribution in both parts of a conductor.

Calculated resistance coefficients for those conductors depending on frequency, are presented in Fig. 7.

The upper pair of curves is for the 680/85 and Martin 1351 conductors, while the smaller resistance coefficients represent 150/25 and Ostrich 300 pair of conductors.

Unexpectedly, probably due to smaller conductivity of ACCR aluminium wires, the resistance coefficient of both ACCR is bigger than of the similar ACSR resistance coefficient.

Much more interesting is the other graphical presentation modus, shown in Fig. 8.

The results presented in Fig. 8 are very interesting. Despite the fact that the conductors are inhomogeneous, this resistance coefficient dependence is a straight line again, as it was for homogeneous conductors. Obviously, in this case, resistance coefficient does not depend on conductors' cross-sections or conductors' conductivity values. The difference between ACSR and ACCR remains, but it is smaller that in previous diagram.

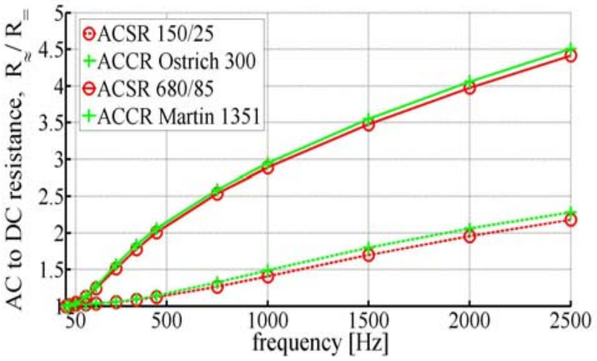

Figure 7. Resistance coefficient as a function of frequency.

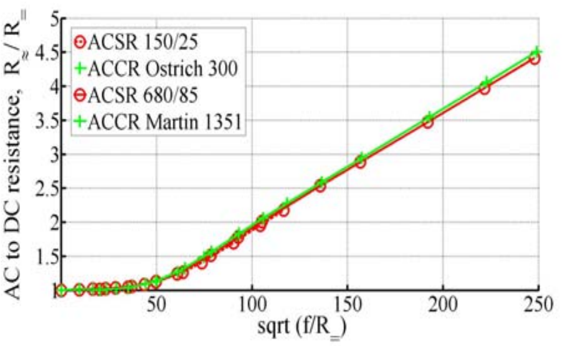

Figure 8. Resistance coefficient as a function of $\sqrt{f / R_=}$ term.

V. Conclusion

An attempt to explore the behavior of inhomogeneous conductors, frequently applied in energy delivery systems, was successfully performed. Current distribution and resistance coefficient were calculated and graphically presented for six types of aluminium conductors, comparing the pairs of ACSR and ACCR conductors with similar cross-sections.

Most of the results were as expected, regarding the current distribution with more or less emphasized skin effect, but the fact that ACSR resistance coefficient is smaller for all calculated frequencies, was a surprise.

Another goal was achieved as well. It was shown that again, one more time, the dependence of resistance coefficient on the term $\sqrt{f / R_=}$ does not depend on conductor's cross-section size, or on conductor's conductivity. For that reason, this functional dependence still is important and convinient for all applications of inhomogenous conductors as well.

All examples were calculated applying COMSOL Multiphysics 3.5a computer program package, which is found one more time as a powerful tool for all electromagnetic fields problems evaluations.

Acknowledgment

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